

DESY 96-052
hep-ph/9604229

Baryogenesis and Lepton number violation

Michael Plümacher*

Deutsches Elektronen-Synchrotron DESY,
Notkestr. 85, D-22603 Hamburg, Germany

Abstract

The cosmological baryon asymmetry can be explained by the nonperturbative electroweak reprocessing of a lepton asymmetry generated in the out-of-equilibrium decay of heavy right-handed Majorana neutrinos. We analyze this mechanism in detail in the framework of a $SO(10)$ -subgroup. We take three right-handed neutrinos into account and discuss physical neutrino mass matrices.

*e-mail: pluemi@x4u2.desy.de

1 Introduction

One of the most striking features of the observable universe is the baryon asymmetry, which is usually expressed as the ratio of the baryon density n_B to the entropy density s . From measurements of the abundances of the light elements one finds¹:

$$Y_B = \frac{n_B}{s} = (0.6 - 1) \times 10^{-10} . \quad (1)$$

With the appearance of grand unified theories (GUTs) it became possible to explain this asymmetry by the baryon number (B) violating decays of Higgs or gauge bosons at the GUT scale. However, these models of baryogenesis are not easily reconciled with inflation. Indeed, a baryon asymmetry present before inflation would be diluted by a huge factor, while the reheating temperature after the inflationary phase is in general too low for these baryogenesis mechanisms to work.

Then, it was realized that anomalous baryon number violating processes are unsuppressed at high temperatures [2]. These so called sphaleron transitions violate $(B + L)$ and conserve $(B - L)$, where L is the lepton number. As sphalerons are in thermal equilibrium for temperatures between $\sim 10^{12}$ GeV and $\sim 10^2$ GeV, they will strongly modify any primordial $(B+L)$ asymmetry. The connection between the baryon asymmetry and a primordial $(B - L)$ asymmetry is given by [3]

$$Y_B = \left(\frac{8N_f + 4N_H}{22N_f + 13N_H} \right) Y_{B-L} , \quad (2)$$

where N_f is the number of fermion families and N_H is the number of Higgs doublets.

The needed primordial $(B - L)$ asymmetry can be realized as a lepton asymmetry generated by the out-of-equilibrium decay of heavy right-handed Majorana neutrinos, as suggested by Fukugita and Yanagida [4]. L is violated by Majorana masses, while the necessary CP violation comes about through phases in the Dirac mass matrix of the neutrinos. In a detailed quantitative analysis Luty showed that the scenario works for a wide range of parameters [5].

In order to generate a lepton asymmetry of the correct order of magnitude, the right-handed neutrinos have to be numerous before decaying. This is only possible if they are in thermal equilibrium at high temperatures. In the original model [4] the right-handed neutrinos were only interacting through Yukawa couplings, which are far too weak to bring the neutrinos into equilibrium at high temperatures. Hence one had to assume an equilibrium distribution for the neutrinos as initial condition in previous analyses.

An appealing way to solve this problem is to study this baryogenesis mechanism in the framework of an extended gauge symmetry, since right-handed neutrinos appear naturally in unified theories based on the gauge groups $SO(10)$ or E_6 . As we shall see, the gauge interactions in which the right-handed neutrinos take part, are strong enough to bring them into thermal equilibrium at high temperatures. Of course, the neutrinos have to be out of equilibrium when

¹For a review and references, see [1]

decaying, i.e. the reaction rates for the gauge interactions have to fall fast enough, so that they cannot significantly reduce the number density of the neutrinos before they decay.

In this paper we investigate this mechanism in the framework of an $SO(10)$ subgroup. After a short discussion of the relevant Boltzmann equations in the next section, we present our model and calculate the needed reaction rates in section 3. In section 4 we solve the Boltzmann equations, first for one and then several heavy neutrino families. We explicitly show that the lepton asymmetry is mainly determined by the lightest of the right-handed neutrinos. Finally we will look at physical mass matrices for the neutrinos coming from an additional abelian gauged family symmetry. These mass matrices give a lepton asymmetry in the right order of magnitude and predict light neutrino masses and mixings of the magnitude needed to explain the solar neutrino deficit and a τ -neutrino mass of a few eV which is needed in the cold-plus-hot dark matter models.

2 Boltzmann Equations

In a quantitative analysis of baryogenesis one can assume Maxwell-Boltzmann statistics [6], so that the equilibrium phase space density of a particle ψ with mass m_ψ is given by

$$f_\psi^{eq}(E_\psi, T) = e^{-E_\psi/T} . \quad (3)$$

The particle density is

$$n_\psi(T) = \frac{g_\psi}{(2\pi)^3} \int d^3p_\psi f_\psi , \quad (4)$$

where g_ψ is the number of internal degrees of freedom. The number of particles Y_ψ in a comoving volume element is given by the ratio of n_ψ and the entropy density s . If the universe expands isentropically Y_ψ is not affected by the expansion of the universe, so that Y_ψ can only be changed by interactions.

We can distinguish between elastic and inelastic scatterings. Elastic scatterings only affect the phase space densities of the particles but not the number densities, whilst inelastic scatterings do change the number densities. If elastic scatterings do occur at a higher rate than inelastic scatterings we can assume kinetic equilibrium, so that the phase space density is [6]

$$f_\psi(E_\psi, T) = \frac{n_\psi}{n_\psi^{eq}} e^{-E_\psi/T} . \quad (5)$$

Consequently the Boltzmann equation describing the evolution of Y_ψ is (cf. [6, 5])

$$\begin{aligned} \frac{dY_\psi}{dz} = & -\frac{z}{sH(m_\psi)} \sum_{a,i,j,\dots} \left[\frac{Y_\psi Y_a \dots}{Y_\psi^{eq} Y_a^{eq} \dots} \gamma^{eq}(\psi + a + \dots \rightarrow i + j + \dots) - \right. \\ & \left. - \frac{Y_i Y_j \dots}{Y_i^{eq} Y_j^{eq} \dots} \gamma^{eq}(i + j + \dots \rightarrow \psi + a + \dots) \right] , \end{aligned} \quad (6)$$

where $z = m_\psi/T$ and $H(m_\psi)$ is the Hubble parameter at $T = m_\psi$.

The right-hand side of eq. (6) describes the interactions in which a ψ particle takes part, where γ^{eq} is the space time density of scatterings in thermal equilibrium. In a dilute gas we only have to take into account decays, two-particle scatterings and the corresponding back reactions. For a decay one finds [5]

$$\gamma_D := \gamma^{eq}(\psi \rightarrow i + j + \dots) = n_\psi^{eq} \frac{K_1(z)}{K_2(z)} \tilde{\Gamma}_{rs} , \quad (7)$$

where K_1 and K_2 are modified Bessel functions and $\tilde{\Gamma}_{rs}$ is the usual decay width in the rest system of the decaying particle. The ratio of the Bessel functions is a time dilatation factor.

If we neglect CP violating effects we have the same reaction density for inverse decays,

$$\gamma_{ID} := \gamma^{eq}(i + j + \dots \rightarrow \psi) = \gamma_D . \quad (8)$$

For two body scattering one has

$$\gamma^{eq}(\psi + a \leftrightarrow i + j + \dots) = \frac{T}{64\pi^4} \int_{(m_\psi+m_a)^2}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right) , \quad (9)$$

where s is the squared center of mass energy and the reduced cross section $\hat{\sigma}(s)$ for the process $\psi + a \rightarrow i + j + \dots$ is related to the usual total cross section $\sigma(s)$ by

$$\hat{\sigma}(s) = \frac{8}{s} \left[(p_\psi \cdot p_a)^2 - m_\psi^2 m_a^2 \right] \sigma(s) . \quad (10)$$

Since we have assumed kinetic equilibrium, contributions from elastic scatterings drop out of eq. (6). Hence we only have to take into account inelastic processes.

3 The model

3.1 Gauge and Yukawa couplings

The 16 plet of $SO(10)$ contains, in addition to the 15 Weyl fermions of one standard model quark-lepton family, a right-handed neutrino ν_R which is a singlet under the standard model gauge group. It is, therefore, natural to embed the baryogenesis mechanism of Fukugita and Yanagida into a $SO(10)$ GUT.

To explain the unification of the coupling constants in $SO(10)$ one needs an intermediate breaking scale v' of the order of 10^{10} to 10^{13} GeV (cf. [7]). The intermediate symmetry could be a left-right-symmetry or a Pati-Salam-symmetry. For simplicity we take the minimal extension of the standard model which is based on the gauge group

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'} . \quad (11)$$

This gauge group cannot explain the gauge coupling unification [7] but it may be regarded as a toy model for the other symmetry groups. We then have the following breaking scheme

$$\begin{aligned} \text{SO}(10) \rightarrow \quad \dots \quad &\longrightarrow \quad \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{Y'} \\ &\xrightarrow{\langle \chi \rangle = v'} \quad \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \\ &\xrightarrow{\langle \phi \rangle = v} \quad \text{SU}(3)_C \times \text{U}(1)_{em} . \end{aligned}$$

Here $\phi = (\varphi^0, \varphi^-)$ is the standard model Higgs doublet and χ is the Higgs boson needed for the breaking of the $\text{U}(1)_{Y'}$. The $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{Y'}$ part of the lagrangian is [8]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} \\ & + i\bar{l}_L \not{D} l_L + i\bar{\nu}_R \not{D} \nu_R + i\bar{e}_R \not{D} e_R + (D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu \chi)^\dagger (D^\mu \chi) \\ & - \left(\bar{l}_L \phi g_\nu \nu_R + \bar{l}_L \tilde{\phi} g_e e_R + \frac{1}{2} \chi \bar{\nu}_R^C h \nu_R + \text{h.c.} \right) , \end{aligned} \quad (12)$$

where we have omitted the quark fields. $l_L = (\nu_L, e_L)$ is the left-handed lepton doublet and ν_R is the right-handed neutrino. The charge conjugated field ν_R^C is defined by $\nu_R^C = C\bar{\nu}_R^T$, where C is the charge conjugation matrix. g_e, g_ν and h are the Yukawa coupling matrices which are responsible for the lepton masses. One can always choose the lepton fields in such a way that g_e and h are diagonal and real.

The covariant derivative has the form

$$D_\mu = \partial_\mu - ig\vec{W}_\mu \cdot \vec{T} - ig'B_\mu Y - ig'\sqrt{\frac{2}{3}}C_\mu Y' , \quad (13)$$

where \vec{W}^μ , B^μ and C^μ are the $\text{SU}(2)_L$, $\text{U}(1)_Y$ and $\text{U}(1)_{Y'}$ gauge fields. $\vec{W}^{\mu\nu}$, $B^{\mu\nu}$ and $C^{\mu\nu}$ are the corresponding field strength tensors. Because of their common origin in the $\text{SO}(10)$ both abelian groups have the same gauge coupling constant g' . From the structure of $\text{SO}(10)$ it follows that $Y' = Y - \frac{5}{4}(B - L)$. Therefore the fermions and the Higgs bosons carry the following Y' -charges

$$\begin{aligned} Y'(l_L) &= \frac{3}{4} & Y'(e_R) &= \frac{1}{4} & Y'(\nu_R) &= \frac{5}{4} \\ Y'(q_L) &= -\frac{1}{4} & Y'(u_R) &= \frac{1}{4} & Y'(d_R) &= -\frac{3}{4} \\ Y'(\phi) &= -\frac{1}{2} & Y'(\chi) &= -\frac{5}{2} . \end{aligned} \quad (14)$$

The most general Higgs potential is

$$V(\chi, \phi) = \mu_1 \phi^\dagger \phi + \mu_2 \chi^\dagger \chi + \frac{1}{2} \lambda_1 (\phi^\dagger \phi)^2 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\chi^\dagger \chi) (\phi^\dagger \phi) , \quad (15)$$

where $\lambda_3 > -\sqrt{\lambda_1 \lambda_2}$, so that the potential is bounded from below. Spontaneous symmetry breaking leads to two massive neutral gauge bosons Z and Z' .

The right-handed neutrinos acquire Majorana masses $M = hv'$ when the $U(1)_{Y'}$ is broken. Additionally the neutrinos get Dirac masses $m_D = vg_\nu$ with $v = 174 \text{ GeV}$ at the electroweak symmetry breaking. This offers the possibility to explain the smallness of the ν_e , ν_μ and ν_τ masses via the seesaw mechanism [9].

Since we want to explain the generation of a lepton asymmetry before the electroweak phase transition, we can take $\langle\phi\rangle = v = 0$, so that all mixing angles vanish. Therefore, all the standard model particles are massless and the additional neutral gauge boson Z' is identical to the vector field C^μ . For $\langle\chi\rangle = v' \neq 0$ the mass of the Z' is

$$m_{Z'} = \frac{5}{\sqrt{3}} g' v' . \quad (16)$$

Since Yukawa couplings are usually small when compared to gauge couplings, one expects that the Majorana masses M_i of the right-handed neutrinos are small compared to the Z' mass.

The Higgs boson H'_0 contained in the field χ gets the mass

$$m_{H'_0} = \sqrt{\lambda_2} v' . \quad (17)$$

To realize the assumed hierarchy $v' \gg v$ one needs $\lambda_2 \geq 2 \cdot 10^{-3}$ [10]. Hence $m_{H'_0}$ is of the order of $m_{Z'}$, so that processes in which a H'_0 takes part are kinematically suppressed at temperatures below $m_{Z'}$. Moreover the Yukawa couplings h of H'_0 were assumed to be small in comparison with the gauge couplings. Therefore the Higgs boson H'_0 can be neglected in the following.

Since we have assumed that the matrix h is real and diagonal the weak eigenstates are equal to the Majorana mass eigenstates,

$$\nu = \nu_L + \nu_L^C \quad \text{and} \quad N = \nu_R + \nu_R^C , \quad (18)$$

where ν and N are four-component Majorana spinors.

3.2 Reaction rates for lepton number violating processes

We are now able to calculate the relevant reaction rates. It appears reasonable to assume a mass hierarchy of the form $M_1 \ll M_2 \ll M_3$ for the right-handed neutrinos, so that the lightest right-handed neutrino, N^1 , will still be in equilibrium when N^2 and N^3 decay. The lepton number violating interactions mediated by N^1 will, therefore, wash out an asymmetry generated by the decays of the neutrinos N^2 and N^3 . Hence a significant lepton asymmetry can only be generated by N^1 -decays and we expect that the temperature at which the asymmetry is generated is of the order of M_1 . Consequently it is useful to relate all the masses and energies to M_1 . We define the following dimensionless quantities

$$a_i := \frac{M_i^2}{M_1^2} , \quad y := \frac{m_{Z'}^2}{M_1^2} \quad \text{and} \quad x := \frac{s}{M_1^2} . \quad (19)$$

The lepton asymmetry is generated by the CP violating decay of the right-handed neutrinos.

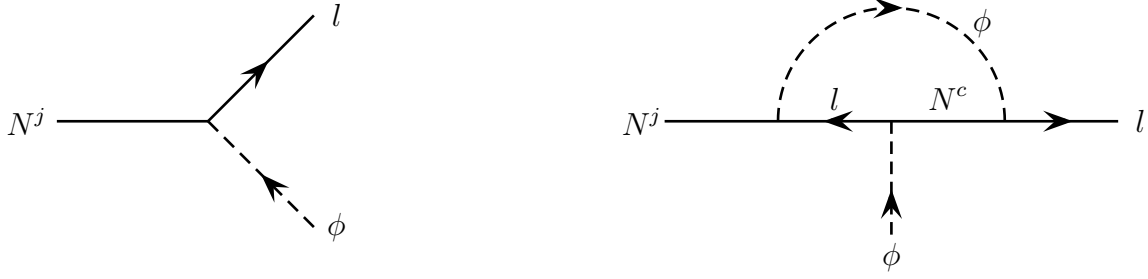


Figure 1: *Right-handed neutrino decay at tree and one-loop level.*

Making use of the relation

$$g_\nu = m_D \frac{1}{v} = m_D \frac{g}{\sqrt{2} M_W}, \quad (20)$$

where $M_W = 80 \text{ GeV}$ is the W boson mass and g is the $\text{SU}(2)$ coupling constant, one finds for the decay width at tree level [4]

$$\tilde{\Gamma}_{Dj} := \tilde{\Gamma}_{rs} (N^j \rightarrow \phi^\dagger + l) + \tilde{\Gamma}_{rs} (N^j \rightarrow \phi + \bar{l}) = \frac{\alpha}{\sin^2 \theta} \frac{M_j}{4} \frac{(m_D^\dagger m_D)_{jj}}{M_W^2}. \quad (21)$$

Here θ is the weak mixing angle. The leading contribution to the CP -asymmetry in the decay of N^j reads

$$\varepsilon_j := \frac{\tilde{\Gamma}_{rs} (N^j \rightarrow \phi^\dagger + l) - \tilde{\Gamma}_{rs} (N^j \rightarrow \phi + \bar{l})}{\tilde{\Gamma}_{rs} (N^j \rightarrow \phi^\dagger + l) + \tilde{\Gamma}_{rs} (N^j \rightarrow \phi + \bar{l})}. \quad (22)$$

It is due to the interference between the tree level decay amplitude and the one loop amplitude shown in fig. 1. One finds²

$$\varepsilon_j = \frac{\alpha}{4 M_W^2 \sin^2 \theta} \frac{1}{(m_D^\dagger m_D)_{jj}} \sum_c \text{Im} \left[(m_D^\dagger m_D)_{jc}^2 \right] f \left(\frac{a_c}{a_j} \right) \quad (23)$$

$$\text{with } f(x) = \sqrt{x} \left[1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right]. \quad (24)$$

From the definition (22) of the CP asymmetry it follows that the reaction densities for decays and inverse decays can be parametrized in the following way

$$\gamma^{eq} (N^j \rightarrow \phi^\dagger + l) = \gamma^{eq} (\phi + \bar{l} \rightarrow N^j) = \frac{1}{2} (1 + \varepsilon_j) \gamma_{Dj} \quad (25)$$

$$\gamma^{eq} (N^j \rightarrow \phi + \bar{l}) = \gamma^{eq} (\phi^\dagger + l \rightarrow N^j) = \frac{1}{2} (1 - \varepsilon_j) \gamma_{Dj} \quad (26)$$

$$\text{with } \gamma_{Dj} := \gamma^{eq} (N^j \rightarrow \phi^\dagger + l) + \gamma^{eq} (N^j \rightarrow \phi + \bar{l}) = \tilde{\Gamma}_{Dj} n_{N^j}^{eq} \frac{K_1(M_j/T)}{K_2(M_j/T)}. \quad (27)$$

Since the CP -asymmetry is a higher order effect we have to take into account other higher order effects as well, especially the L violating lepton Higgs scattering $l + \phi^\dagger \rightarrow \bar{l} + \phi$ shown in

²The results quoted in the literature differ from our result by a factor 8 (ref. [5]), 2 (ref. [7]) or 18 (ref. [4])

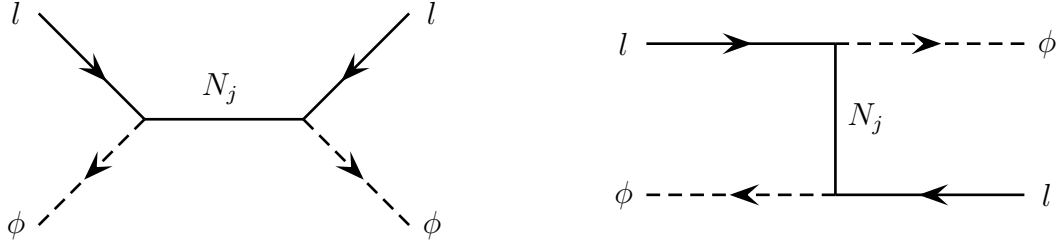


Figure 2: *Lepton number violating lepton Higgs scattering*

fig. 2 with a right-handed neutrino as intermediate state. As shown in ref. [6] this diagram is necessary to avoid the generation of a lepton asymmetry in thermal equilibrium.

However, the scatterings with a real intermediate neutrino were already taken into account, because they can be described by an inverse decay followed by a decay. Therefore, we have to remove the contributions from physical intermediate states from the reduced cross section[6].

In [5] the flavour structure of this diagram was neglected, i.e. only the contribution from the lightest right-handed neutrino N^1 was taken into account. We will take into consideration two right-handed neutrinos, N^1 and N^2 . When discussing our results we will see that the third neutrino can indeed be neglected. The reduced cross section is then given by:

$$\begin{aligned} \hat{\sigma}'_N(s) = & \frac{\alpha^2}{\sin^4 \theta} \frac{2\pi}{M_W^4} \frac{1}{x} \left\{ \sum_{j=1}^2 a_j (m_D^\dagger m_D)_{jj}^2 \left[\frac{x}{a_j} + \frac{2x}{D_j(x)} + \frac{x^2}{2D_j^2(x)} - \left(1 + 2\frac{x+a_j}{D_j(x)} \right) \ln \left(\frac{x+a_j}{a_j} \right) \right] \right. \\ & + 2\sqrt{a_1 a_2} \operatorname{Re} \left[(m_D^\dagger m_D)_{12}^2 \right] \left[\frac{x}{D_1(x)} + \frac{x}{D_2(x)} + \frac{x^2}{2D_1(x)D_2(x)} \right. \\ & \left. \left. - \frac{(x+a_1)(x+a_1-2a_2)}{D_2(x)(a_1-a_2)} \ln \left(\frac{x+a_1}{a_1} \right) - \frac{(x+a_2)(x+a_2-2a_1)}{D_1(x)(a_2-a_1)} \ln \left(\frac{x+a_2}{a_2} \right) \right] \right\}. \quad (28) \end{aligned}$$

The prime denotes that we have subtracted the contributions from real intermediate states and

$$\frac{1}{D_j(x)} := \frac{x - a_j}{(x - a_j)^2 + a_j c_j}, \quad \text{with } c_j := \left(\frac{\tilde{\Gamma}_{Dj}}{M_1} \right)^2, \quad (29)$$

is the off-shell part of the propagator.

There are some other L violating processes mediated by a right-handed neutrino in the t -channel, like the scattering $l + l \rightarrow \phi + \phi$. The reduced cross section for this process, which was neglected in [5], is

$$\begin{aligned} \hat{\sigma}_{N,t}(s) = & \frac{2\pi\alpha^2}{M_W^4 \sin^4 \theta} \left\{ \sum_{j=1}^2 a_j (m_D^\dagger m_D)_{jj}^2 \left[\frac{x}{x+a_j} + \frac{1}{x+2a_j} \ln \left(\frac{x+a_j}{a_j} \right) \right] \right. \\ & \left. + \operatorname{Re} \left[(m_D^\dagger m_D)_{12}^2 \right] \frac{\sqrt{a_1 a_2}}{(a_1 - a_2)(x + a_1 + a_2)} \left[(x + 2a_1) \ln \left(\frac{x+a_2}{a_2} \right) - (x + 2a_2) \ln \left(\frac{x+a_1}{a_1} \right) \right] \right\}. \quad (30) \end{aligned}$$

The same result is valid for the process $\bar{l} + \bar{l} \rightarrow \phi^\dagger + \phi^\dagger$ and the back reactions.

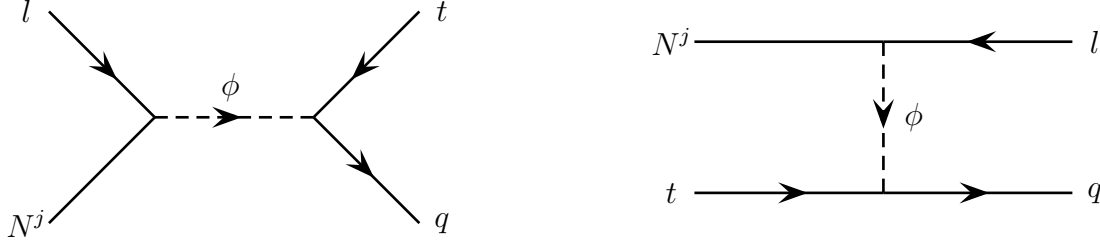


Figure 3: *Lepton number violating scatterings mediated by a standard model Higgs boson in the s- or t-channel*

The Yukawa couplings of the standard model are very small and can be neglected, with the exception of the term responsible for the top quark mass m_t . Thus we have to take into account the L violating interactions of a right-handed neutrino with a top-quark. There are s- and t-channel contributions, shown in fig. 3. The reduced cross section for the s-channel process $N^j + l \rightarrow \bar{t} + q$ is [5]

$$\hat{\sigma}_{\phi,s}^j(s) = \frac{3\pi\alpha^2 m_t^2}{M_W^4 \sin^4 \theta} (m_D^\dagger m_D)_{jj} \left(\frac{x - a_j}{x} \right)^2. \quad (31)$$

One finds the same result for the process $N^j + \bar{l} \rightarrow t + \bar{q}$ and the corresponding back reactions.

To get a well defined result for the t-channel contribution one has to introduce a Higgs mass m_ϕ . Keeping only the leading contributions for $m_\phi \rightarrow 0$ one finds [5]

$$\hat{\sigma}_{\phi,t}^j(s) = \frac{3\pi\alpha^2 m_t^2}{M_W^4 \sin^4 \theta} (m_D^\dagger m_D)_{jj} \left[\frac{x - a_j}{x} + \frac{a_j}{x} \ln \left(\frac{x - a_j + y'}{y'} \right) \right], \quad (32)$$

with the dimensionless squared Higgs mass $y' := m_\phi^2/M_1^2$. Since these processes have only a small influence on the generated asymmetry, the results are very insensitive to the value of m_ϕ . In the following we use $m_\phi = 800 \text{ GeV}$.

3.3 Reaction rates for lepton number conserving processes

The lepton number violating processes are too weak to bring the right-handed neutrinos into equilibrium at high temperatures. Therefore, one has to consider lepton number conserving processes, which were neglected up to now, like the N^j pair creation and pair annihilation. The dominating contribution comes from the Z' exchange in the s-channel shown in fig. 4a. The total reduced cross section for the processes $f + \bar{f} \leftrightarrow N^j + N^j$, where f is a standard model fermion, and $\phi + \phi^\dagger \leftrightarrow N^j + N^j$ is

$$\hat{\sigma}_{Z'}(s) = \frac{4225\pi}{216} \frac{\alpha^2}{\cos^4 \theta} \frac{\sqrt{x}}{(x - y)^2 + yc} (x - 4a_j)^{3/2}. \quad (33)$$

To get a well defined result we have introduced the Z' decay width into the propagator,

$$c := \left(\frac{\tilde{\Gamma}_{Z'}}{M_1} \right)^2 \quad \text{with} \quad \tilde{\Gamma}_{Z'} = \frac{\alpha m_{Z'}}{\cos^2 \theta} \left[\frac{25}{18} \sum_i \left(\frac{y - 4a_i}{4y} \right)^{3/2} \theta(y - 4a_i) + \frac{169}{144} \right]. \quad (34)$$

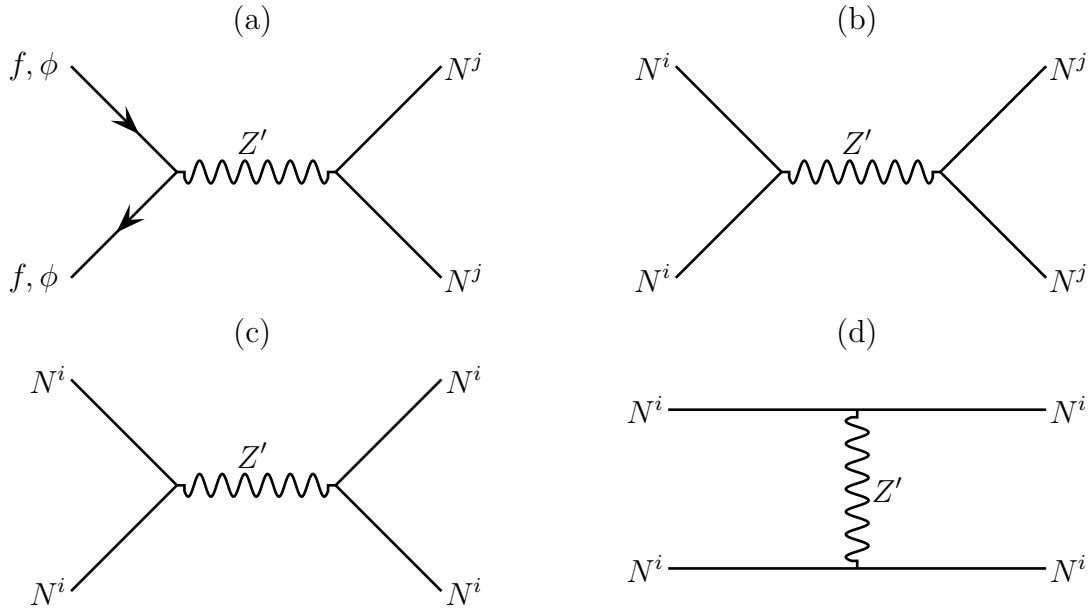


Figure 4: *Lepton number conserving processes:*

- (a) N^j pair creation
- (b) process $N^i + N^i \rightarrow N^j + N^j$ ($i \neq j$)
- (c,d) elastic scattering $N^i + N^i \rightarrow N^i + N^i$

Since we will have different heavy neutrino flavours we have to consider transitions between right-handed neutrinos like $N^i + N^i \leftrightarrow N^j + N^j$ ($i \neq j$) with an intermediate Z' in the s -channel, shown in fig. 4b. The reduced cross section for this process is

$$\hat{\sigma}_{N^i N^j}(s) = \frac{625\pi}{216} \frac{\alpha^2}{\cos^4 \theta} \left\{ \frac{1}{x} \frac{(x - 4a_i)^{3/2} (x - 4a_j)^{3/2}}{(x - y)^2 + yc} + 12 \frac{a_i a_j}{y^2} \sqrt{\frac{x - 4a_i}{x}} \sqrt{\frac{x - 4a_j}{x}} \right\}. \quad (35)$$

Finally we have to look at the elastic processes to check that the assumed kinetic equilibrium is a good approximation. For this purpose we have considered the process $N^i + N^i \rightarrow N^i + N^i$. The dominating contribution to this transition comes from Z' exchange in the s -channel (fig. 4c) and in the t - and u -channel (fig. 4d). The reduced cross section for this process reads

$$\begin{aligned} \hat{\sigma}_{el}(s) = & \frac{625\pi}{72} \frac{\alpha^2}{\cos^4 \theta} \left\{ \frac{1}{3x} \frac{(x - 4a_i)^3}{(x - y)^2 + yc} + \right. \\ & + \frac{x - 4a_i}{xy^2} \frac{y(x - 4a_i)^2 + 2(y - 2a_i)^3 + (8a_i^2 + 3y^2)x - 4a_i(y^2 + ya_i + 4a_i^2)}{x - 4a_i + y} + \\ & \left. + \frac{1}{xy} \frac{(3y - 4a_i)(x - 4a_i)^2 + (3x - 20a_i)y^2 + 2y(y^2 + 8a_i^2)}{x - 4a_i + 2y} \ln \left(\frac{y}{x - 4a_i + y} \right) \right\}. \quad (36) \end{aligned}$$

4 Results

4.1 Constraints on the parameters

Before solving the Boltzmann equations we will try to constrain the parameters by looking at the reaction rates. The neutrinos have to be out of equilibrium when decaying, i.e. they have to decouple before decaying. Therefore, the decay rate Γ_D has to be smaller than the Hubble parameter H at temperatures $T \approx M_1$. This gives the following constraint [11, 12]

$$\Gamma_D(T = M_1) < 3H(T = M_1) \quad \Leftrightarrow \quad \tilde{m}_1 := \frac{(m_D^\dagger m_D)_{11}}{M_1} < 9 \cdot 10^{-3} \text{ eV} . \quad (37)$$

\tilde{m}_1 is the mass of the lightest neutrino, if m_D is approximately diagonal.

Furthermore the pair annihilation rate $\Gamma_{Z'}$ for the neutrinos has to be smaller than $3H$ at $T \approx M_1$. This gives a lower bound on the Z' mass [5, 10]

$$\Gamma_{Z'}(T = M_1) < 3H(T = M_1) \quad \Leftrightarrow \quad m_{Z'} > 2 \cdot 10^{11} \text{ GeV} \left(\frac{M_1}{10^{10} \text{ GeV}} \right)^{3/4} . \quad (38)$$

On the other hand the L asymmetry must be generated before the electroweak phase transition, because we need the sphalerons which are no longer in equilibrium at temperatures below 100 GeV. Since the electroweak phase transition takes place 10^{-12} s after the big bang, the lifetime of the right-handed neutrinos has to be smaller than 10^{-12} s. This yields the following constraint

$$(m_D^\dagger m_D)_{11} > (20 \text{ eV})^2 \left(\frac{10^{10} \text{ GeV}}{M_1} \right) . \quad (39)$$

Finally the reaction rates Γ_N and $\Gamma_{N,t}$ for the lepton number violating scatterings mediated by the right-handed neutrinos must not wash out the generated lepton asymmetry at low temperatures. This gives the following condition [13]

$$\Gamma_N(T = M_1) < 3H(T = M_1) \quad \Leftrightarrow \quad \sum_j \tilde{m}_j^2 < (7 \text{ eV})^2 \left(\frac{10^{10} \text{ GeV}}{M_1} \right) . \quad (40)$$

The sum over \tilde{m}_j^2 can be interpreted as sum over the squared masses of the light neutrinos, if m_D is approximately diagonal.

We have shown the different reaction rates and the Hubble parameter in fig. 5, where we have chosen the following parameters

$$\tilde{m}_1 = 10^{-3} \text{ eV} , \quad M_1 = 10^{10} \text{ GeV} \quad \text{and} \quad m_{Z'} = 2 \cdot 10^{11} \text{ GeV} . \quad (41)$$

The reaction rates have the correct behaviour. $\Gamma_{Z'}$ is much bigger than $3H$ at temperatures $T \gg M_1$, so that the neutrinos N^1 come into equilibrium at high temperatures, while $\Gamma_{Z'}$ is small enough at $T \approx M_1$ to avoid the lepton number conserving pair annihilation of the neutrinos. Furthermore one can see that the reaction rates taken into account in previous analyses [4, 5], i.e. Γ_D , Γ_{ID} , Γ_N , $\Gamma_{\phi,s}$ and $\Gamma_{\phi,t}$ are far too low to bring the neutrinos into thermal equilibrium at high temperatures.

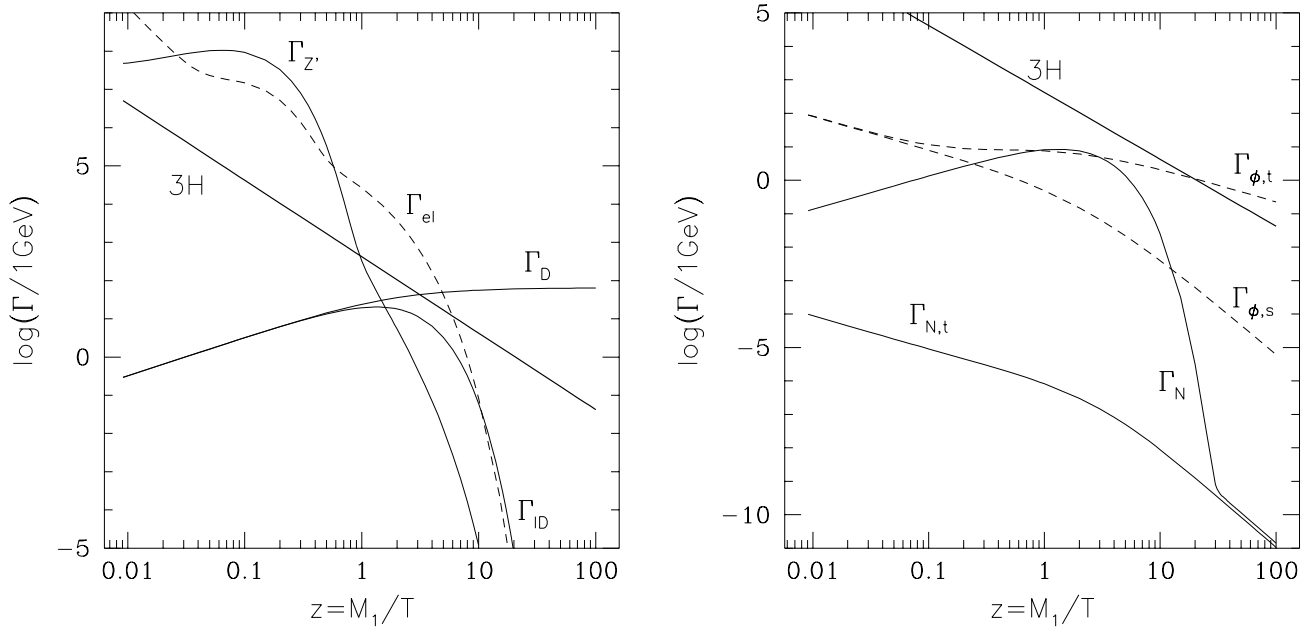


Figure 5: Comparison of different reaction rates with the Hubble parameter H . $\Gamma_{Z'}$ is the pair annihilation rate, Γ_D the decay rate, Γ_{ID} the reaction rate for inverse decays, Γ_N and $\Gamma_{N,t}$ for lepton number violating processes with an intermediate N^1 , $\Gamma_{\phi,s}$ and $\Gamma_{\phi,t}$ for the interaction of an incoming N^1 with a top-quark and Γ_{el} for the elastic N^1 -scattering.

Since the decay rate Γ_D is smaller than $3H$ for $T \approx M_1$, the neutrinos have enough time to deviate from the thermal equilibrium, so that the out-of-equilibrium condition is fulfilled. Moreover the reaction rates Γ_N and $\Gamma_{N,t}$ for the lepton number violating interactions and Γ_{ID} for inverse decays are so small that the L asymmetry cannot be erased at temperatures $T < M_1$.

Finally we have plotted the reaction rate Γ_{el} for the elastic scatterings $N^1 + N^1 \leftrightarrow N^1 + N^1$. As one can see Γ_{el} is much bigger than $\Gamma_{Z'}$ in the relevant temperature region $T \approx M_1$, so that one can assume kinetic equilibrium.

4.2 Decay of the lightest heavy neutrino

For simplicity we begin by neglecting the decays of the right-handed neutrinos N^2 and N^3 . Using the formalism of section 2 and the reaction rates calculated in section 3 we can immediately write down the Boltzmann equations. If we assume that all the standard model particles are in thermal equilibrium, the Boltzmann equations for the N^1 number and for the $(B - L)$ asymmetry are

$$\begin{aligned} \frac{dY_{N^1}}{dz} &= -\frac{z}{sH(M_1)} \left\{ \left(\frac{Y_{N^1}}{Y_{N^1}^{eq}} - 1 \right) [\gamma_{D1} + 2\gamma_{\phi,s}^1 + 4\gamma_{\phi,t}^1] + \left[\left(\frac{Y_{N^1}}{Y_{N^1}^{eq}} \right)^2 - 1 \right] \gamma_{Z'} \right\}, \quad (42) \\ \frac{dY_{B-L}}{dz} &= -\frac{z}{sH(M_1)} \left\{ \left[\frac{1}{2} \frac{Y_{B-L}}{Y_l^{eq}} + \varepsilon_1 \left(\frac{Y_{N^1}}{Y_{N^1}^{eq}} - 1 \right) \right] \gamma_{D1} + \right. \end{aligned}$$

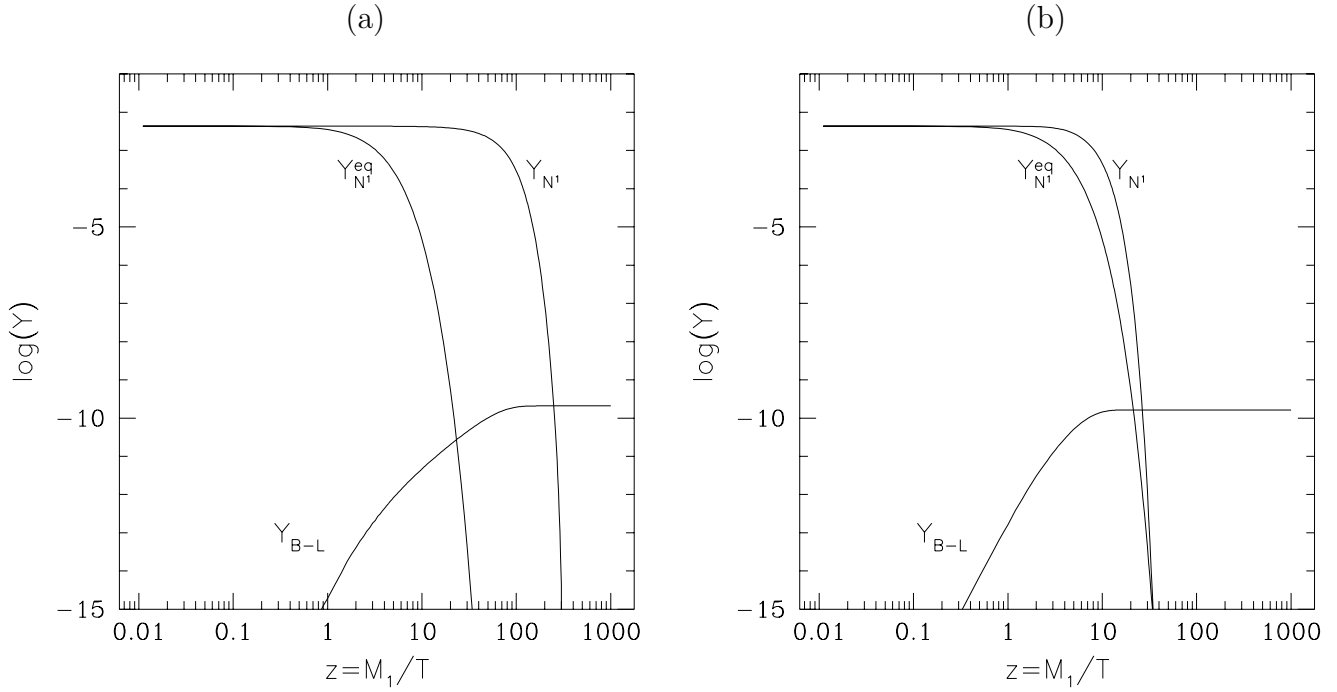


Figure 6: *Typical solutions of the Boltzmann equations.*

$$+ \frac{Y_{B-L}}{Y_l^{eq}} \left[2\gamma_N + 2\gamma_{N,t} + 2\gamma_{\phi,t}^1 + \frac{Y_{N^1}}{Y_{N^1}^{eq}} \gamma_{\phi,s}^1 \right] \Bigg\} . \quad (43)$$

These equations have to be solved numerically. We start at $T \gg M_1$ with the initial conditions

$$Y_{N^1} = Y_{N^1}^{eq} \quad \text{and} \quad Y_{B-L} = 0 \quad (44)$$

and follow the evolution of these quantities down to low temperatures. We want to emphasize that the results are independent of the initial conditions (44), because the reaction rates are so high that the neutrinos are rapidly driven into equilibrium at high temperatures while any primordial $(B - L)$ asymmetry is washed out. Typical results are shown in fig. 6, where we have used the following parameters

$$M_1 = 10^{10} \text{ GeV} \quad m_{Z'} = 10^{13} \text{ GeV} \quad \varepsilon_1 = -5 \cdot 10^{-8} \quad (45)$$

$$\text{and} \quad \tilde{m}_1 := \frac{(m_D^\dagger m_D)_{11}}{M_1} = \begin{cases} 10^{-6} \text{ eV} & \text{for fig. 6a} \\ 10^{-4} \text{ eV} & \text{for fig. 6b} . \end{cases} \quad (46)$$

One recognizes very clearly the departure from thermal equilibrium and the generation of the asymmetry in the correct order of magnitude. If the neutrinos decay at very low temperatures $T \ll M_1$, the back reactions and the lepton number violating scatterings can be neglected, so that we expect a $(B - L)$ asymmetry of the order [1]

$$Y_{B-L} \approx -\varepsilon_1 Y_{N^1}(T = M_1) = 2.1 \cdot 10^{-10} . \quad (47)$$

This is exactly the result of fig. 6a, while one has

$$Y_{B-L} = 1.6 \cdot 10^{-10} \quad (48)$$

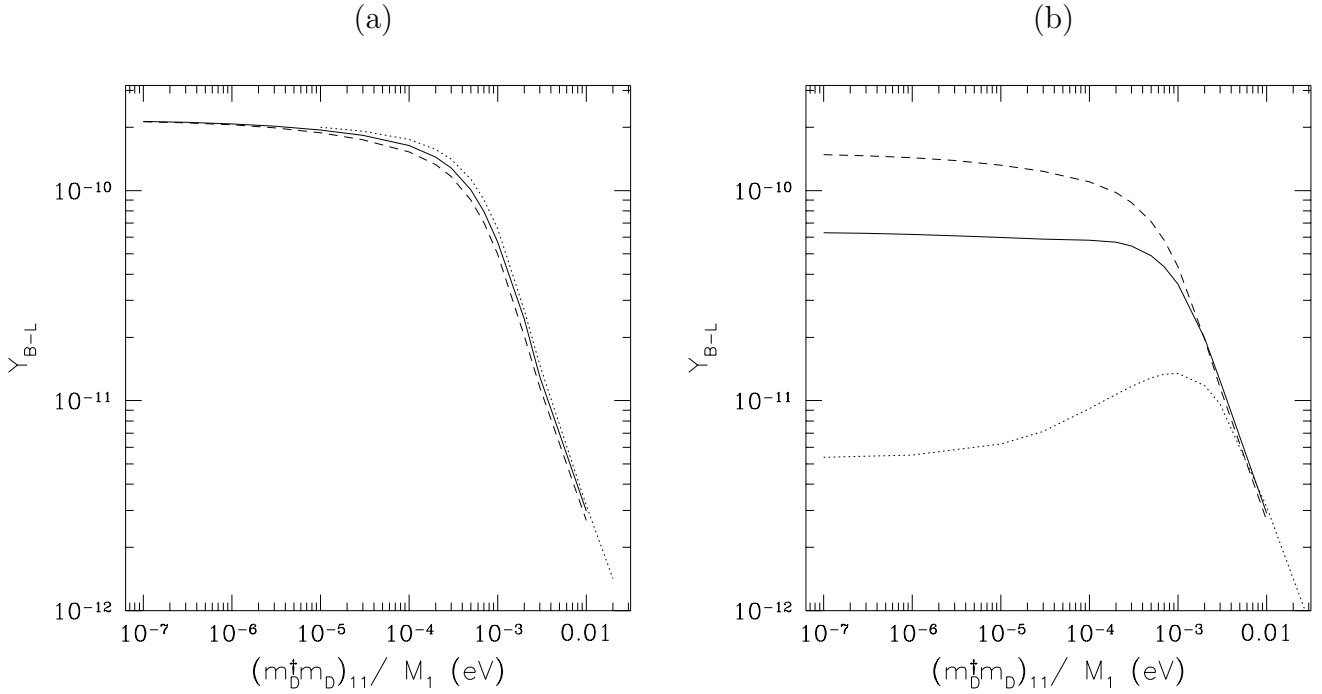


Figure 7: *The generated $(B-L)$ asymmetry for $m_{Z'} = 10^3 M_1$ (a) and $m_{Z'} = 10 M_1$ (b) and for $M_1 = 10^8 \text{ GeV}$ (dotted line), $M_1 = 10^{10} \text{ GeV}$ (solid line) and $M_1 = 10^{12} \text{ GeV}$ (dashed line).*

in fig. 6b. This means that the inverse decays and the lepton number violating scatterings already erase a part of the asymmetry. A further increase in $(m_D^\dagger m_D)_{11}$ would intensify the dissipative processes, so that the final asymmetry would be even lower.

One gets almost the same results if one varies the parameters and keeps the ratios $y = (m_{Z'}/M_1)^2$ and \tilde{m}_1 and the CP asymmetry ε_1 constant. In fig. 7a we have plotted the generated asymmetry as a function of \tilde{m}_1 for $y = 10^6$ and $\varepsilon_1 = -5 \cdot 10^{-8}$ for the neutrino masses $M_1 = 10^{12} \text{ GeV}$, 10^{10} GeV and 10^8 GeV .

First of all one notices that the condition (37) delimitates quite accurately the range of \tilde{m}_1 in which the generation of the $(B-L)$ asymmetry is possible with a reasonable choice for the CP asymmetry.

In the second place it is remarkable that the asymmetry depends almost entirely on \tilde{m}_1 and not on $(m_D^\dagger m_D)_{11}$ and M_1 separately. This becomes clear if one looks at the right-hand side of the Boltzmann equation (43). Indeed the decay term depends only on \tilde{m}_1 ,

$$\frac{z}{sH(M_1)} \gamma_{D1} \propto \tilde{m}_1. \quad (49)$$

The same proportionality holds for $\gamma_{\phi,s}$ and $\gamma_{\phi,t}$. On the other hand one finds

$$\frac{z}{sH(M_1)} \gamma_N \propto \tilde{m}_1^2 M_1 \quad \text{and} \quad \frac{z}{sH(M_1)} \gamma_{N,t} \propto \tilde{m}_1^2 M_1 \quad (50)$$

for the lepton number violating scatterings. Since these terms can erase a lepton asymmetry one would expect that the asymmetry is falling with M_1 . This is exactly what one can observe in fig. 7a.

Up to now we have always chosen a large Z' mass, $m_{Z'} = 10^3 M_1$, to ensure that pair annihilation processes do not influence the evolution of Y_{N1} at temperatures $T < M_1$. As a counter-example we have repeated our calculations with a lower Z' mass, $m_{Z'} = 10 M_1$. The results are summarized in fig. 7b. In contrast to fig. 7a one recognizes a strong dependence of the asymmetry on M_1 . This comes about because of the pair annihilation term in the Boltzmann equation (42), which is inversely proportional to M_1 ,

$$\frac{z}{sH(M_1)} \gamma_{Z'} \propto \frac{1}{M_1}. \quad (51)$$

Therefore, the pair annihilation processes should be more effective at low neutrino masses M_1 . This explains the dependence of the curves in fig. 7b on M_1 .

Fig. 7b may further be used to confirm the lower bound (38) on the Z' mass. One can generate the required asymmetry for $M_1 = 10^{12}$ and 10^{10} GeV but not for $M_1 = 10^8$ GeV, because the condition (38) is only fulfilled for the first two values of M_1 .

The curve for $M_1 = 10^8$ GeV in fig. 7b has another interesting property. Up to now the generated asymmetry was always monotonically falling with \tilde{m}_1 , while Y_{B-L} has now a maximum for $\tilde{m}_1 \approx 10^{-3}$ eV. The reason is, that for this value of \tilde{m}_1 the neutrinos decay before they can be depleted by pair annihilation processes, while the reaction rate for inverse decays is still too small to erase the asymmetry. The evolution of the asymmetry is hampered by pair annihilation processes at lower values of \tilde{m}_1 and by dissipating processes like inverse decays at higher values of \tilde{m}_1 .

4.3 Decays of two heavy neutrinos

Now we can refine our results by adding a second family of right-handed neutrinos. The Boltzmann equations are straightforward generalizations of the equations (42) and (43)

$$\begin{aligned} \frac{dY_{B-L}}{dz} = & -\frac{z}{sH(M_1)} \left\{ \sum_{j=1}^2 \left[\frac{1}{2} \frac{Y_{B-L}}{Y_l^{eq}} + \varepsilon_j \left(\frac{Y_{Nj}}{Y_{Nj}^{eq}} - 1 \right) \right] \gamma_{Dj} \right. \\ & \left. + \frac{Y_{B-L}}{Y_l^{eq}} [2\gamma_N + 2\gamma_{N,t}] + \frac{Y_{B-L}}{Y_l^{eq}} \sum_{j=1}^2 \left[2\gamma_{\phi,t}^j + \frac{Y_{Nj}}{Y_{Nj}^{eq}} \gamma_{\phi,s}^j \right] \right\}, \end{aligned} \quad (52)$$

$$\begin{aligned} \frac{dY_{N1}}{dz} = & -\frac{z}{sH(M_1)} \left\{ \left(\frac{Y_{N1}}{Y_{N1}^{eq}} - 1 \right) [\gamma_{D1} + 2\gamma_{\phi,s}^1 + 4\gamma_{\phi,t}^1] + \right. \\ & \left. + \left[\left(\frac{Y_{N1}}{Y_{N1}^{eq}} \right)^2 - 1 \right] \gamma_{Z'} + \left[\left(\frac{Y_{N1}}{Y_{N1}^{eq}} \right)^2 - \left(\frac{Y_{N2}}{Y_{N2}^{eq}} \right)^2 \right] \gamma_{N^1 N^2} \right\}, \end{aligned} \quad (53)$$

$$\begin{aligned} \frac{dY_{N2}}{dz} = & -\frac{z}{sH(M_1)} \left\{ \left(\frac{Y_{N2}}{Y_{N2}^{eq}} - 1 \right) [\gamma_{D2} + 2\gamma_{\phi,s}^2 + 4\gamma_{\phi,t}^2] + \right. \\ & \left. + \left[\left(\frac{Y_{N2}}{Y_{N2}^{eq}} \right)^2 - 1 \right] \gamma_{Z'} + \left[\left(\frac{Y_{N2}}{Y_{N2}^{eq}} \right)^2 - \left(\frac{Y_{N1}}{Y_{N1}^{eq}} \right)^2 \right] \gamma_{N^1 N^2} \right\}, \end{aligned} \quad (54)$$

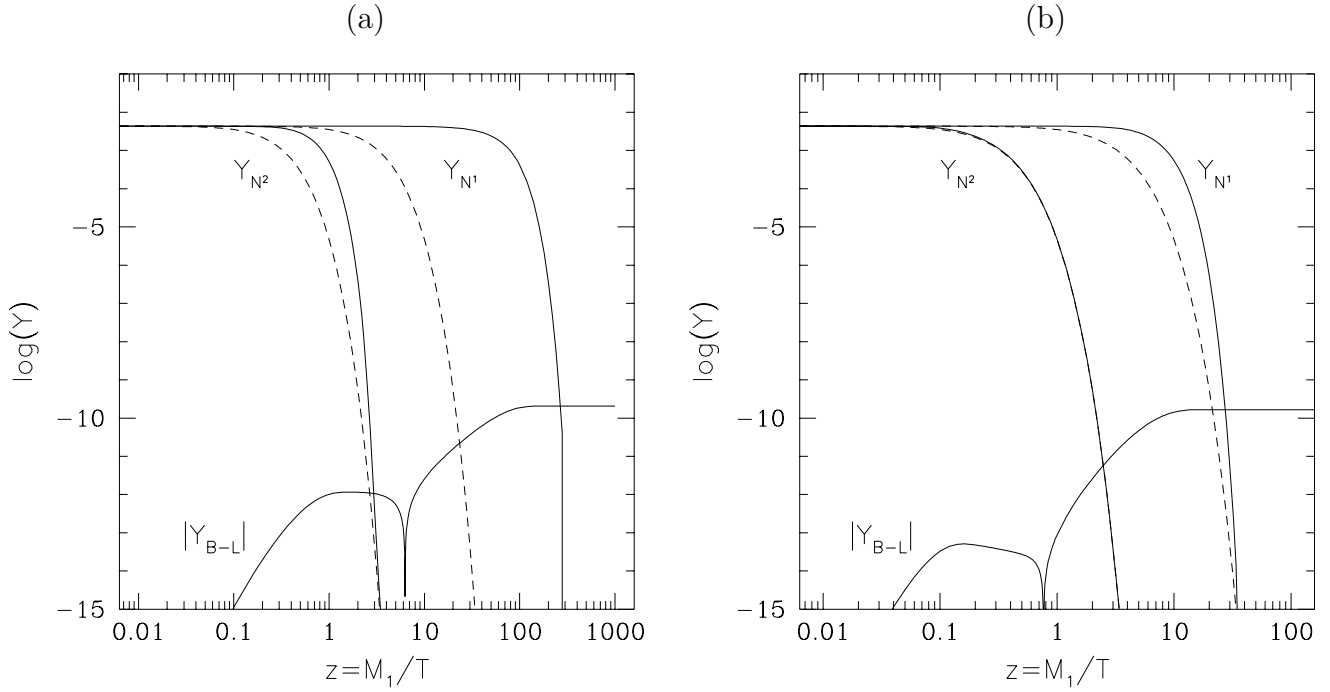


Figure 8: *Typical solutions of the Boltzmann equations with two neutrino families. The equilibrium distributions are represented by dashed lines.*

where $\gamma_{N^1 N^2}$ is the reaction density for the N^1 - N^2 -transitions described by the reduced cross section (35).

The constraints which were derived in section 4.1 remain valid, but the parameters for the two heavy neutrinos are not always independent of each other. For the CP asymmetries one has for example:

$$\varepsilon_1 \propto \text{Im} \left[\left(m_D^\dagger m_D \right)_{12}^2 \right] \quad \text{and} \quad \varepsilon_2 \propto \text{Im} \left[\left(m_D^\dagger m_D \right)_{21}^2 \right]. \quad (55)$$

Since

$$\left(m_D^\dagger m_D \right)_{12} = \left(m_D^\dagger m_D \right)_{21}^*, \quad (56)$$

the two CP asymmetries have a different sign. Therefore, the asymmetries generated in the decays of N^1 and N^2 will also have a different sign, so that they can cancel each other if the neutrinos are mass degenerate. But this should not be a problem if the neutrinos have hierarchical masses like the charged fermions, because the asymmetry generated by the heavier neutrinos is more affected by the dissipative processes and because the mass dependence of ε_1 and ε_2 ensures that $|\varepsilon_2|$ is smaller than $|\varepsilon_1|$ if $M_2 > M_1$. Analogous phenomena have already been observed in other models [14].

Typical results of the numerical integration of the Boltzmann equations are shown in fig. 8. In both cases we have chosen

$$M_1 = 10^{10} \text{ GeV}, \quad M_2 = 10^{11} \text{ GeV}, \quad M_{Z'} = 10^{13} \text{ GeV} \quad \text{and} \quad \text{Re} \left[\left(m_D^\dagger m_D \right)_{12}^2 \right] = 0. \quad (57)$$

The results are almost independent of the choice $\text{Re} \left[\left(m_D^\dagger m_D \right)_{12}^2 \right] = 0$, because this parameter only appears in interference terms between the different diagrams contributing to the lepton Higgs scatterings which have only a small influence on the generated asymmetry.

Furthermore we have for fig. 8a

$$\begin{aligned} \tilde{m}_1 = \frac{\tilde{m}_2}{100} = 10^{-6} \text{ eV} \quad \text{and} \quad \frac{\text{Im} \left[\left(m_D^\dagger m_D \right)_{12}^2 \right]}{M_1^2} = 7.7 \cdot 10^{-8} (\text{eV})^2 \\ \Leftrightarrow \quad \varepsilon_1 = -5 \cdot 10^{-8} \quad \text{and} \quad \varepsilon_2 = 3.7 \cdot 10^{-10} , \end{aligned} \quad (58)$$

and for fig. 8b we have

$$\begin{aligned} \tilde{m}_1 = \frac{\tilde{m}_2}{100} = 10^{-4} \text{ eV} \quad \text{and} \quad \frac{\text{Im} \left[\left(m_D^\dagger m_D \right)_{12}^2 \right]}{M_1^2} = 7.7 \cdot 10^{-6} (\text{eV})^2 \\ \Leftrightarrow \quad \varepsilon_1 = -5 \cdot 10^{-8} \quad \text{and} \quad \varepsilon_2 = 3.7 \cdot 10^{-10} . \end{aligned} \quad (59)$$

Therefore, we have chosen exactly the same parameters for N^1 as in the one-family calculations of fig. 6, so that we can compare the results. First of all one sees that the asymmetry changes sign at $z \approx 6$ and $z \approx 0.8$ as we had predicted it at the beginning of this section. But the influence of the second neutrino family on the final asymmetry is only very small and we recover the results (47) and (48). Consequently one can always neglect the heavier neutrino families if one has a pronounced mass hierarchy.

4.4 Physical mass matrices

Up to now we have only verified that the parameters of the theory can be chosen in such a way, that they allow the generation of the requested asymmetry. However, in a physical theory the parameters have to comply with some other conditions. First of all the mass scale of the right-handed neutrinos and the Z' boson have to be related to the intermediate breaking scale v' . Next the predicted light neutrino masses have to be consistent with the experimental bounds. We will focus on the following conditions:

1. The solar neutrino deficit can be explained by the Mikheyev-Smirnov-Wolfenstein (MSW) effect, if the ν_e and ν_μ masses and their mixing angle fulfill the following conditions (cf. [7])

$$\delta m^2 := m_{\nu_\mu}^2 - m_{\nu_e}^2 = (0.3 - 1) \cdot 10^{-5} (\text{eV})^2 , \quad (60)$$

$$\sin^2 2\theta_{e\mu} = 0.003 - 0.012 ; \quad (61)$$

2. a ν_τ mass of a few eV is needed in the cold-plus-hot dark matter models.

A realistic pattern of masses and mixings of the known fermions can be obtained based on a gauged abelian family symmetry broken below the unification scale [15]. This model predicts

the following mass matrices for the up and down quarks (suppressing unknown phases and factors which are assumed to be of order 1)

$$M_u \approx \begin{pmatrix} \epsilon^{|2+6a|} & \epsilon^{|3a|} & \epsilon^{|1+3a|} \\ \epsilon^{|3a|} & \epsilon^2 & \epsilon \\ \epsilon^{|1+3a|} & \epsilon & 1 \end{pmatrix} m_t \quad \text{and} \quad M_d \approx \begin{pmatrix} \bar{\epsilon}^{|2+6a|} & \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|1+3a|} \\ \bar{\epsilon}^{|3a|} & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^{|1+3a|} & \bar{\epsilon} & 1 \end{pmatrix} m_b, \quad (62)$$

with $a = \alpha_3/(\alpha_2 - \alpha_3)$, where α_i is the family charge of the i -th generation quarks. One obtains a good agreement with the measured values of the quark masses and mixings for

$$a = 1 \quad \text{and} \quad \sqrt{\epsilon} = \bar{\epsilon} = 0.23. \quad (63)$$

Similarly the leptons of the i -th generation have the family charge a_i , and therefore the mass matrix of the charged leptons reads

$$M_l \approx \begin{pmatrix} \bar{\epsilon}^{|8-2b|} & \bar{\epsilon}^3 & \bar{\epsilon}^{|4-b|} \\ \bar{\epsilon}^3 & \bar{\epsilon}^{|2(1-b)|} & \bar{\epsilon}^{|1-b|} \\ \bar{\epsilon}^{|4-b|} & \bar{\epsilon}^{|1-b|} & 1 \end{pmatrix} m_\tau \quad \text{for integer } b = \frac{\alpha_2 - a_2}{\alpha_2 - \alpha_3}. \quad (64)$$

The known masses of the charged leptons can be explained for $b = 0$. M_l is diagonalized by an orthogonal matrix R_l . If one neglects higher order terms in $\bar{\epsilon}$, R_l is given by

$$R_l \approx \begin{pmatrix} 1 & \delta_{e\mu} & \mathcal{O}(\bar{\epsilon}^4) \\ -\delta_{e\mu} & 1 & \mathcal{O}(\bar{\epsilon}) \\ -\mathcal{O}(\bar{\epsilon}^4) & -\mathcal{O}(\bar{\epsilon}) & 1 \end{pmatrix} \quad \text{with } \delta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} = \mathcal{O}(\bar{\epsilon}^2). \quad (65)$$

This model also predicts Dirac masses m_D for the neutrinos and Majorana masses M for the right-handed neutrinos [16],

$$m_D = \begin{pmatrix} \bar{\epsilon}^{16} & \bar{\epsilon}^6 & \bar{\epsilon}^8 \\ \bar{\epsilon}^6 & \bar{\epsilon}^4 & \sigma \bar{\epsilon}^2 \\ \bar{\epsilon}^8 & \sigma \bar{\epsilon}^2 & \rho \end{pmatrix} m_{\nu_3} \quad \text{and} \quad M = \begin{pmatrix} \bar{\epsilon}^8 & \tau \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \tau \bar{\epsilon}^3 & \zeta \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix} M_3, \quad (66)$$

where we have introduced four factors ρ , σ , τ and ζ of order 1, which will be used to fix the physical parameters. As we are working with mass eigenstates N^i we have to transform m_D and M into a basis in which M is diagonal. Up to higher order terms in $\bar{\epsilon}$ the matrix R_M which diagonalizes M is

$$R_M = \begin{pmatrix} 1 & \bar{\epsilon}^2 & \bar{\epsilon}^4 \\ -\bar{\epsilon}^2 & 1 & \bar{\epsilon} \\ -\bar{\epsilon}^4 & -\bar{\epsilon} & 1 \end{pmatrix}, \quad (67)$$

which yields the following masses for the heavy neutrinos

$$M' := R_M^T M R_M \approx \begin{pmatrix} -2\tau \bar{\epsilon}^5 & 0 & 0 \\ 0 & (\zeta - 1)\bar{\epsilon}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_3. \quad (68)$$

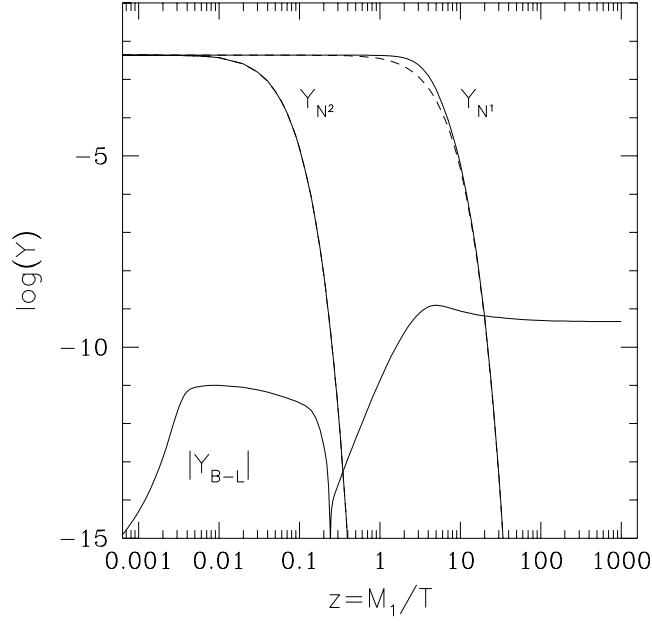


Figure 9: *Solutions of the Boltzmann equations with physical mass matrices. The equilibrium distributions are represented by dashed lines.*

Therefore the mass ratios of the right-handed neutrinos are

$$a_2 = \left(\frac{M_2}{M_1}\right)^2 \approx \frac{(\zeta - 1)^2}{4\tau^2\bar{\epsilon}^6} \quad \text{and} \quad a_3 = \left(\frac{M_3}{M_1}\right)^2 \approx \frac{1}{4\tau^2\bar{\epsilon}^{10}}. \quad (69)$$

The Dirac mass matrix in the new basis is

$$m'_D := m_D R_M \approx \begin{pmatrix} -\bar{\epsilon}^8 & \bar{\epsilon}^6 & \bar{\epsilon}^7 \\ -(1+\sigma)\bar{\epsilon}^6 & -\sigma\bar{\epsilon}^3 & \sigma\bar{\epsilon}^2 \\ -(\rho+\sigma)\bar{\epsilon}^4 & -(\rho-\sigma\bar{\epsilon})\bar{\epsilon} & \rho \end{pmatrix} m_{\nu_3}. \quad (70)$$

The mass parameters which appear in the decay widths are thus given by

$$\tilde{m}_1 = \frac{(m'^{\dagger}_D m'_D)_{11}}{M_1} \approx \frac{(\rho+\sigma)^2\bar{\epsilon}^2}{2\tau} \frac{m_{\nu_3}^2}{M_3} \quad \text{and} \quad \tilde{m}_2 = \frac{(m'^{\dagger}_D m'_D)_{22}}{M_2} \approx \frac{(\rho-\sigma\bar{\epsilon})^2}{|\zeta-1|} \frac{m_{\nu_3}^2}{M_3}. \quad (71)$$

Up to now we have neglected all the phase factors that we need for CP violation. In a Majorana mass matrix the number of physical phases for the n generation case is $n(n-1)/2$ (cf. [7]). They can be chosen in such a way, that the squared non diagonal elements of $m'^{\dagger}_D m'_D$ are purely imaginary. Then the CP asymmetries are

$$\varepsilon_1 \approx \tau \left[\rho^2 + \frac{(\rho-\sigma\bar{\epsilon})^2}{|\zeta-1|} \right] \left(\frac{m_{\nu_3}}{3.6 \cdot 10^4 \text{ GeV}} \right)^2 \quad \text{and} \quad \varepsilon_2 \approx \rho^2 |\zeta-1| \left(\frac{m_{\nu_3}}{5.6 \cdot 10^3 \text{ GeV}} \right)^2. \quad (72)$$

Now we have to specify the parameters ρ , σ , τ and ζ . A good choice is

$$\rho = 0.9, \quad \sigma = -1, \quad \tau = -\frac{1}{4} \quad \text{and} \quad \zeta = 2. \quad (73)$$

The mass scales are approximately fixed by their breaking scale,

$$m_{\nu_3} = 50 \text{ GeV} , \quad M_3 = 5 \cdot 10^{11} \text{ GeV} \quad \text{and} \quad M_{Z'} = 10^{12} \text{ GeV} . \quad (74)$$

Then the Majorana neutrinos N^1 and N^2 have the masses

$$M_1 = 3.1 \cdot 10^8 \text{ GeV} \quad \text{and} \quad M_2 = 2.6 \cdot 10^{10} \text{ GeV} . \quad (75)$$

For the coupling parameters (71) and the CP asymmetries (72) one finds

$$\tilde{m}_1 = 8 \cdot 10^{-4} \text{ eV} \quad \text{and} \quad \tilde{m}_2 = 6.4 \text{ eV} , \quad (76)$$

$$\varepsilon_1 = -1.3 \cdot 10^{-6} \quad \text{and} \quad \varepsilon_2 = 4.6 \cdot 10^{-5} . \quad (77)$$

The corresponding solutions of the Boltzmann equations are shown in fig. 9. The generated asymmetry,

$$Y_{B-L} = 5 \cdot 10^{-10} , \quad (78)$$

is even bigger than requested, but it is always possible to reduce Y_{B-L} by choosing the phase factors in an appropriate way. One finds similar results for other values of the parameters ρ , σ , τ and ζ .

Finally we have to check if these matrices can predict the desired light neutrino masses and mixings. The Majorana masses of the light neutrinos are given by the seesaw formula [9],

$$m_\nu = m_D \frac{1}{M} m_D^T \approx \begin{pmatrix} -2\bar{\epsilon}^{15} & (\sigma-1)\bar{\epsilon}^9 & -\rho\bar{\epsilon}^7 \\ (\sigma-1)\bar{\epsilon}^9 & \sigma^2\tau\bar{\epsilon}^4 & \rho\sigma\tau\bar{\epsilon}^2 \\ -\rho\bar{\epsilon}^7 & \rho\sigma\tau\bar{\epsilon}^2 & \rho^2\tau \end{pmatrix} \frac{1}{\tau} \frac{m_{\nu_3}^2}{M_3} . \quad (79)$$

The eigenvalues of this matrix are

$$m_{\nu_e} = 8.9 \cdot 10^{-7} \text{ eV} , \quad m_{\nu_\mu} = 1.9 \cdot 10^{-3} \text{ eV} \quad \text{and} \quad m_{\nu_\tau} = 2.2 \text{ eV} . \quad (80)$$

These values fulfill the MSW condition (60) and the ν_τ mass is in the correct range for the cold-plus-hot dark matter models. m_ν is approximately diagonalized by the following matrix,

$$R_\nu \approx \begin{pmatrix} 1 & 2\bar{\epsilon}^3 & \bar{\epsilon}^5 \\ -2\bar{\epsilon}^3 & 1 & (\sigma/\rho)\bar{\epsilon}^2 \\ -\bar{\epsilon}^5 & -(\sigma/\rho)\bar{\epsilon}^2 & 1 \end{pmatrix} . \quad (81)$$

The leptonic analogon of the Cabbibo-Kobayashi-Maskawa (CKM) matrix is

$$V_l = (R_\nu)^{-1} R_l = \begin{pmatrix} 1 & \delta_{e\mu} - 2\bar{\epsilon}^3 & -\bar{\epsilon}^4 \\ -\delta_{e\mu} + 2\bar{\epsilon}^3 & 1 & \bar{\epsilon} \\ \bar{\epsilon}^4 & -\bar{\epsilon} & 1 \end{pmatrix} . \quad (82)$$

Therefore the ν_e - ν_μ mixing angle complies with the MSW condition (61)

$$\sin^2 2\theta_{e\mu} = 0.009 . \quad (83)$$

5 Conclusions

We have seen that the cosmic baryon asymmetry can be explained by the lepton number violating decays of heavy Majorana neutrinos combined with the anomalous electroweak ($B+L$) violation. The lepton asymmetry generated in our model is independent of the initial conditions on the heavy neutrino density, which was not the case in a previous analysis of this mechanism [5]. This is a consequence of the new gauge interaction which we have introduced, and which is related to the spontaneous breaking of lepton number.

We have also considered the decay of more than one heavy neutrino and have seen that the generated asymmetry is determined by the properties of the lightest heavy neutrino, if the right-handed neutrinos have a pronounced mass hierarchy.

By performing explicit calculations we could show that the generation of a lepton asymmetry is possible at every temperature between the intermediate breaking scale and the electroweak breaking scale. Furthermore we have checked that neutrino mass matrices may explain the generation of the asymmetry and low energy neutrino phenomenology at the same time.

In supersymmetric models of inflation the reheating temperature has to be lower than 10^5 to 10^8 GeV to solve the gravitino problem [17, 18]. Therefore in these models baryogenesis has to take place at relatively low temperatures. Since this is possible in our model, a supersymmetric generalization should be viable.

Acknowledgments

I would like to thank W. Buchmüller, who suggested this investigation, for continuous support and encouragement.

References

- [1] E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley, Redwood City, CA, 1990)
- [2] N. S. Manton, Phys. Rev. **D 28** (1983) 2019
F. R. Klinkhammer and N. S. Manton, Phys. Rev. **D 30** (1984) 2212
V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B 155** (1985) 36
- [3] S. Yu. Khlebnikov and M. E. Shaposhnikov Nucl. Phys. **B 308** (1988) 885
- [4] M. Fukugita and T. Yanagida, Phys. Lett. **B 174** (1986) 45
- [5] M. A. Luty, Phys. Rev. **D 45** (1992) 455

- [6] E. W. Kolb and S. Wolfram, Nucl. Phys. **B 172** (1980) 224, Erratum: Nucl. Phys. **B 195** (1982) 542
- [7] M. Fukugita and T. Yanagida, *Physics of Neutrinos*, in: *Physics and Astrophysics of Neutrinos*, eds. M. Fukugita and A. Suzuki, (Springer-Verlag, Tokyo, 1994)
- [8] W. Buchmüller, C. Greub and P. Minkowski, Phys. Lett. **B 267** (1991) 395
- [9] H. Fritzsch and P. Minkowski, Phys. Lett. **B 62** (1976) 72
T. Yanagida, in: *Workshop on unified Theories*, KEK report 79-18 (1979) p. 95
M. Gell-Mann et al., in: *Supergravity*, Proc. of the Stony Brook Workshop 1979, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315
- [10] K. Enqvist and I. Vilja, Phys. Lett. **B 299** (1993) 281
K. Enqvist, K. Kainulainen and I. Vilja, Nucl. Phys. **B 403** (1993) 749
- [11] W. Fischler, G. F. Giudice, R. G. Leigh and S. Paban, Phys. Lett. **B 258** (1991) 45
- [12] W. Buchmüller and T. Yanagida, Phys. Lett. **B 302** (1993) 240
- [13] M. Fukugita and T. Yanagida, Phys. Rev. **D 42** (1990) 1285
J. A. Harvey and M. S. Turner, Phys. Rev. **D 42** (1990) 3344
- [14] J. A. Harvey, E. W. Kolb, D. B. Reiss and S. Wolfram, Nucl. Phys. **B 201** (1982) 16
- [15] L. Ibáñez and G. G. Ross, Phys. Lett. **B 332** (1994) 100
- [16] H. Dreiner, G. K. Leontaris, S. Lola, G. G. Ross and C. Scheich, Nucl. Phys. **B 436** (1995) 461
- [17] G. G. Ross and S. Sarkar, Nucl. Phys. **B 461** (1996) 597
S. Sarkar, talk presented at the International Europhysics Conference on High Energy Physics (HEP 95), Brussels, Belgium, 27 Jul - 2 Aug 1995. Preprint hep-ph/9510369
- [18] B. A. Campbell, S. Davidson and K. A. Olive, Nucl. Phys. **B 399** (1993) 111